Momentum-Field Interactions Beyond Standard Quadratic Optomechanics

Sina Khorasani
Laboratory of Photonics and Quantum Measurements, École Polytechnique Fédérale de Lausanne CH-1015, Lausanne, Switzerland; Email: sina.khorasani@epfl.ch

Faculty of Physics, University of Vienna (*Universität Wien*)*, Vienna, Austria*
November 22

The standard theory of quantum optomechanics [1,2] is founded on an interaction Hamiltonian of the form

$$\mathbb{H}_{int} = \frac{1}{2}\left[\hat{P}^2 + \omega^2(\hat{q})\hat{Q}^2\right] = \frac{1}{2}\left[\hat{P}^2 + \frac{\pi^2}{\hat{q}^2}\hat{Q}^2\right],$$

in which $\hat{q}$, $\hat{P}$, and $\hat{Q}$ are respectively the position of the massive object, and the first and second quadratures of the electromagnetic radiation. What is less clear at first, is that both $\hat{P}$ and $\hat{Q}$ are actually functions of $\omega(\hat{q})$, and therefore the entire interaction takes place only between the optical field and mirror displacement $\hat{q}$. Hence, we have $\mathbb{H}_{int} \equiv \mathbb{H}_{int}(\hat{q})$. It is after Taylor's expansion of the operators in terms of mirror displacement $\hat{x} = \hat{q} - L$ (with L being the unperturbed cavity length) and subsequently dropping the new resulting non-interacting terms, that the well-known optomechanical interaction $\mathbb{H}_{\text{OM}} = -\hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger)$ is obtained, where the phononic contribution $\hat{b} + \hat{b}^\dagger$ represents $\hat{q}$ within a constant of proportionality. Similarly, various orders of interaction Hamiltonians become $\mathbb{H}_n = \hbar(-1)^n g_{n-1} \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger)^n$, while $n = 0$ adds up to the optical field's self-energy, $n = 1$ stands for $\mathbb{H}_{\text{OM}}$, and $n = 2$ for the quadratic interaction, etc.

I) In the first part of my talk, it is shown that not all plausible interactions take place between the cavity photons $\hat{a}^\dagger \hat{a}$ and mirror position $\hat{q}$. But it is rather expected that other types of interactions between light and mirror momentum $\hat{p}$ could also happen [3]. However, such interactions begin to occur only at the quadratic order and continue into the higher-orders, while rapidly decaying to zero with increasing order. Furthermore, under most experimental conditions, even the lowest order new quadratic correction to the interaction Hamiltonian is negligible, which is having the form $\Delta\mathbb{H}_2 = \beta \hat{p}^2 \hat{Q}^2$ with β being some constant. Interestingly, relativistic corrections are shown to also have exactly the same type and sign of contribution at the quadratic order.

While the relativistic corrections appear to be vanishingly small and always negligible under practical limits, the quadratic momentum-field is not. It is expected that certain class of experiments could probe the possible existence of such type of interactions. These conditions and higher-order correction terms resulting from momentum-field interactions are discussed.

II) In the second part of my talk, a new mathematical scheme [4] is uncovered, which is able to solve the nonlinear Langevin equations of nonlinear quantum mechanical interactions, by defining higher-order operators and then linearizing the resulting operator equations unto an expanded basis. The method allows study of nonlinear quantum effects and their stability, without the conventional linearization and mean-field approximations, and instead employing the existing mathematical toolbox of linear analysis.

[1] S. K. Law, "Interaction Between a Moving Mirror and Radiation Pressure: A Hamiltonian Formulation," *Phys. Rev. A* **51**, 2537 (1995).
[2] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, *Cavity Optomechanics*, Springer, Berlin, 2014.
[3] S. Khorasani, "Higher-Order Interactions in Quantum Optomechanics: Revisiting the Theoretical Foundations," *Appl. Sci.* **7**, 656 (2017).
[4] S. Khorasani, "Higher-Order Interactions in Quantum Optomechanics: Analytical Solution of Nonlinearity," arxiv: 1702.04982 (2017).